Buoyancy Guide

National Precast Concrete Association

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Section 1 - Buoyancy Guide

1.1 Intro

Buoyancy is defined as the tendency of a fluid to exert a supporting upward force on a body placed in a fluid (i.e., a liquid or a gas). The fluid can be a liquid, as in the case of a boat floating on a lake, or the fluid can be a gas, as in a helium-filled balloon floating in the atmosphere. An elementary application of buoyancy can be seen when trying to push an empty water bottle downward in a sink full of water. When applying a downward force to the water bottle from your hand, the water bottle will stay suspended in place. But, as soon as you remove your hand from the water bottle, the water bottle will float to the surface. The buoyant force on the object plays a factor whether a given object will sink or float in a fluid.

People have been aware of objects floating or sinking in a fluid since the beginning of time. It wasn’t officially documented and conceptually grasped until Archimedes (287-212 B.C.), established the theory of flotation, and defined the buoyancy principle. Apparently (while in his bath) Archimedes realized that submerged objects always displace fluid upward (water in the bath rose while he was submerged). Then with that observation, he concluded that this force (buoyant) must be equal to the weight of the displaced fluid. Archimedes then went on to state that a solid object would float if the density of the solid object was less than the density of the fluid and vice versa.

![Figure 1: Archimedes' Principle](image)

So, why is buoyancy an important factor in the design of an underground concrete structure? The simple answer is that the buoyant force created by water needs to be resisted to prevent a structure from floating or shifting upward. Essentially a concrete structure will not float if the sum of the vertical downward forces (gravitational, W) is greater than the vertical upward force (buoyant, FB). When applying this principle to a structure below grade, it can be said that if the buoyant force (FB) is greater than the mass of the structure and the combined mass of soil surcharges and objects contained within the structure, the structure will float. This guide will help direct the user how to apply basic buoyancy principles through a series of standard calculations and procedures. The result will determine if their underground concrete structure will resist buoyant forces.
Section 2- Determining Water Table Levels

2.1 Review Plans, Specifications, Soils Reports and Boring Logs

When designing an underground precast concrete structure, it is necessary to know what structure to make as well as what its intended use is. Typically, a contractor who is in need of a precast structure will come to precasters with details on what they need and give design requirements and information on the underground conditions. But not always do they inform precasters about every detail, especially job site conditions and problems in the construction area. Job site conditions and underground conditions are vital pieces of information needed for the design calculations to optimize the performance of the structure in the installed condition and to prevent floatation. So, how does the design engineer determine when there could be a potential problem with the jobsite conditions and with floatation?

First off, the design engineer should do his/her the research to review and investigate the plans, specifications and soils reports to gain more insight about the project and the underground conditions. After obtaining the design requirements and specifications for the structural design the design engineer should obtain extensive information on the soils and underground conditions. One of the first factors that must be determined when analyzing an area, in which the concrete structure will be placed below grade, is the water table,(i.e."groundwater level") (WL). Obtaining this information will help the designers identify areas where flotation could potentially be a problem and areas where flotation will most likely not be a factor in the design. So, how can one determine the water table level in the project area?

The design engineer should check the soils report to obtain more information on the area. The soils report is most likely the most reliable source of information since it's based on a study of the jobsites' conditions. If there isn't a soils report, core drilling may be necessary. By core drilling in the vicinity of the project, the depth of the water level from grade can be determined. It should be noted that groundwater levels identified on boring reports are only a snapshot and may not account for seasonal variations. Another possible source of information would be from local well drillers. They should maintain records of water table levels in areas which they drill. If there isn't any of the above information available, you may want to ask your local excavation contractors, since they have firsthand experience with groundwater in the area. If you're out of options and you can't find out any of the above information, consider a design for water levels at grade, even if flooding in that area is not common. This is considered a conservative approach by many design engineers. A conservative design approach may offset unnecessary and unforeseen cost when sufficient information about the soil/site conditions is unavailable.

After the water table/level has been determined and it is known that there will most likely not be a problem with buoyancy or flotation issues, the designer can focus in on maximizing the structure without buoyant forces being exerted on the structure. In most cases flotation will not be a problem in areas of the country without groundwater (parts of Texas, Arizona, Nevada, etc.), and where the groundwater is below the anticipated depth of the structure. The fact that the buoyancy force (FB) exists presupposes that the water table at the site is believed to be some distance above the lowest point of the installed structure. But, if your structure is to be placed above
the groundwater level (according to the sites’ water table), less concern is needed. On the other hand, areas where flotation causes potential problems are regions where the water level is at grade (valleys, ocean shores), and areas where groundwater is present below grade, at time of installation (before soil has had a chance to become a solid mass).

2.2 Be Aware of Seasonal and Regional Variations
The water table is the upper level of an underground surface in which the soil is saturated with water. The water table fluctuates both with the seasons and from year to year because it is affected by climatic variations and by the amount of precipitation used by vegetation. It also is affected by withdrawing excessive amounts of water from wells or by recharging them artificially (5). The design engineer should make certain to factor the water table level seasonal and regional fluctuations into the design of an underground precast concrete structure. This will ensure that the underground structure will not float or shift upward from a water table level miscalculation.

2.3 Err on the Conservative Side
If there are no soils reports or previous information available on the water table levels and fluctuations (seasonal and regional), most design engineers will design the structure on the conservative side. This will ensure that the structure will be able to withstand seasonal and regional fluctuations.

Designing on the conservative side refers to designing a structure with the water level at grade, even if flooding in that area is not common. A conservative design approach may contribute to offsetting unnecessary and unforeseen cost when sufficient information about the soil/site conditions is unavailable. Therefore, overdesigning a structure should be kept to a minimum since this would add substantial costs to production.
Section 3- Computing Downward (Gravity) Forces

After the water table level has been determined, the design engineer needs to look at computing all the downward forces that will be acting on the structure. All vertical downward forces are caused by gravitational effects, which need to be calculated in the design of an underground structure in order to determine if the total downward forces (gravitational, WT) are greater than the upward force (buoyant FB). The total downward force (WT) is calculated by the summation of all downward vertical forces (W).

\[ W_T = W_1 + W_2 + W_3 + W_4 \ldots \]

Depending on the design of the underground structure, the total vertical downward forces (WT) may or may not be the same for all applications. In a conservative approach, the design of underground structures assumes that the water table, at the specific site, is at grade. With this being the case, it is essential that all vertical downward forces (WT) be accounted for, to ensure that the structure will not float (WT > FB). For an underground structure, designed for a worst-case scenario, the following vertical downward forces (W) need to be accounted for:

- Weight of all walls and slabs
- Weight of soil on slabs
- Weight of soil on shelf (or shelves)
- Weight of equipment inside structure (permanent)
- Weight of inverts inside structure
- Friction of soil to soil
- Additional concrete added inside structure
- Weight of reinforcing steel

As noted earlier, not all underground structures are the same, and therefore some of the listed vertical downward forces (W) above may not be included in the summation of total vertical downward force (WT).

3.1 Specific Gravity and Density

The density of an object is calculated as its weight divided by the volume. In the imperial system, the density of an object is expressed as lbs/ft³. The density unit for the metric system is usually expressed as kg/m³. Specific Gravity (SG) is the ratio of an object’s density to the density of water. SG values have no units – regardless if the calculations are metric or imperial.

For example:
Concrete with a density (“unit weight“) of 150 lbs/ft³. Water has a density of 62.4 lbs/ft³.
The SG of that concrete is 150 lbs/ft³ / 62.4 lbs/ft³ = 2.4.

3.1 Weight of Concrete

The total weight of the concrete that makes up the structure can account for a large portion of the downward gravitational force. In the design of an underground precast concrete structure a certain thickness of the walls and slabs must be implemented, whereas other products, such as HDPE have thinner wall sections and tend to float up out of the ground when they are placed (if not properly anchored). With a specific gravity of 2.40, precast concrete
products resist the buoyant forces associated with underground construction. In comparison, fiberglass has a specific gravity of 1.86 and high-density polyethylene (HDPE) has a specific gravity of 0.97.

The weight of the walls and slabs can be determined by calculating the volume \( V \), in \( \text{ft}^3 \) (m\(^3\)), of the underground structure and multiplying the volume by the density \( \rho \), in \( \text{lbs/ft}^3 \) (kg/m\(^3\)), of the concrete. The calculation for determining the weight of walls and slabs for a rectangular or round structure are listed below:

**RECTANGULAR**

\[
W_{\text{WALL}} = l \times w \times t \times \rho \quad (3.1.1)
\]

\( W_{\text{WALL}} = \text{weight of the wall segment (lbs.)} \)

\( l = \text{length of the wall segment (ft.)} \)

\( w = \text{width of the wall segment (ft.)} \)

\( t = \text{thickness of wall segment (ft.)} \)

\( \rho = \text{density of concrete (lbs/ft}^3) \)

\[
W_{\text{SLAB}} = l \times h \times t \times \rho \quad (3.1.3)
\]

\( W_{\text{SLAB}} = \text{weight of slab (lbs)} \)

\( l = \text{length of slab segment (ft)} \)

\( h = \text{height of slab segment (ft.)} \)

\( t = \text{thickness of slab segment (ft.)} \)

\( \rho = \text{density of concrete (lbs/ft}^3) \)

**ROUND**

\[
W_{\text{BARREL}} = 0.25\pi \times (OD^2 - ID^2) \times d \times \rho \quad (3.1.2)
\]

\( W_{\text{BARREL}} = \text{weight of manhole barrel section (lbs.)} \)

\( OD = \text{outside diameter of barrel (ft.)} \)

\( ID = \text{inside diameter of barrel (ft.)} \)

\( d = \text{height of section (ft.)} \)

\( \rho = \text{density of concrete (lbs/ft}^3) \)

\[
W_{\text{SLAB}} = 0.25\pi d^2 \times t \times \rho \quad (3.1.4)
\]

\( W_{\text{SLAB}} = \text{weight of round slab (lbs)} \)

\( d = \text{diameter of round slab (ft.)} \)

\( t = \text{thickness of slab segment (ft.)} \)

\( \rho = \text{density of concrete (lbs/ft}^3) \)

As a general guideline, the range of values for heavy, normal, and lightweight concrete densities are as follows:

**Heavy-weight**: 165-330 (lbs/ft\(^3\))

**Normal-weight**: 140-150 (lbs/ft\(^3\))

**Lightweight**: 90-115 (lbs/ft\(^3\))
If the structure has an extended base, the additional concrete weight must be calculated and included to determine the entire weight of the structure. The calculations for determining the weight of concrete for rectangular and round structures with an extended base are as follows:

### RECTANGULAR

\[
W_{\text{SHELF}} = 2(l + 2s + w) \times B_s \times \rho \times s \quad (3.1.5)
\]

**WSHELF** = weight of concrete shelf extension (lbs)

\[
l = \text{length of rectangular structure (ft.)}
\]

\[
s = \text{width of shelf (ft.)}
\]

\[
w = \text{width of rectangular structure (ft.)}
\]

\[
B_s = \text{thickness of base slab (ft.)}
\]

\[
\rho = \text{density of concrete (lbs./ft}^3\text{)}
\]

### ROUND

\[
W_{\text{SHELF}} = 0.25\pi \times (OD_{\text{SHELF}}^2 - OD^2) \times B_s \times \rho \quad (3.1.6)
\]

**WSHELF** = weight of concrete shelf extension (lbs)

\[
OD_{\text{SHELF}} = \text{outside diameter of shelf (ft.)}
\]

\[
OD = \text{outside diameter of barrel (ft)}
\]

\[
B_s = \text{thickness of base slab (ft.)}
\]

\[
\rho = \text{density of concrete (lbs./ft}^3\text{)}
\]

### 3.2 Subtract for Openings

When calculating the weight of an underground precast concrete structure, openings must be accounted for and subtracted from the walls and slabs of the structure. The following equations are used to determine the approximate weight of the concrete removed due to the openings.

### Weight of a Subtracted Cylinder Opening

\[
W_{\text{CYLINDER}} = 0.25\pi \times OD^2 \times t \times \rho \quad (3.2.1)
\]

**W_{\text{CYLINDER}}** = weight of circular cut out (lbs.)

\[
OD = \text{outside diameter of cut out (ft.)}
\]

\[
t = \text{wall thickness (ft.)}
\]

\[
\rho = \text{density of concrete (lbs./ft}^3\text{)}
\]

### Weight of a Subtracted Rectangular Opening

\[
W_{\text{REC}} = l \times w \times t \times \rho \quad (3.2.2)
\]

**W_{\text{REC}}** = weight of rectangular opening (lbs.)

\[
l = \text{length of rectangular opening (ft.)}
\]

\[
w = \text{width of rectangular opening (ft.)}
\]

\[
t = \text{wall thickness (ft.)}
\]

\[
\rho = \text{density of concrete (lbs./ft}^3\text{)}
\]
3.3 Weight of Earth Fill

The weight of the soil acting vertically on the slab top is determined by multiplying the surface area of the structure by the depth of the fill and the density of the soil. The density of the soil is dependent on conditions such as geology, compaction, and moisture content. Soil above the water table has moisture contents that can range from 0% to 100% of the absorption values; soils below the water table are usually 100% saturated. The dry soil density can be determined by the soils report or engineering knowledge and judgement by the designer. To determine the submerged soil density is more complicated and will require the designer to also have knowledge for the soil specific gravity, Gs, of the soils under consideration. That with the additional assumption that the submerged soils have a saturation of 100% will provide the submerged soil density relationship, as shown in Equation 3.3.1.

\[
\rho_s = \left[1 - (1/G_s)\right] \times \rho_d \quad (3.3.1)
\]

Where:

\( \rho_s \) = submerged soil density (lbs./ft\(^3\))

\( G_s \) = soil specific gravity

\( \rho_d \) = dry soil density (lbs./ft\(^3\))

If all the required soil parameters are not readily known to accurately calculate \( \rho_s \), a quick and conservative approach to develop a simplified submerged soil density, \( \rho_{ss} \), is to subtract the water density, \( \rho_w \), from the dry soil density, \( \rho_d \), as shown in Equation 3.3.2.

\[
\rho_{ss} = \rho_d - \rho_w \quad (3.3.2)
\]

This simplified method will provide a submerged soil density values slightly less in density than the more rigorous calculation of \( \rho_s \), and consequently provide a more conservative analysis. This method is utilized within the sample problems below. Should the designer have available all the geotechnical soil parameters to confidently calculate the actual submerged soil density, that value can be utilized within the methodology shown within equation 3.3.1 above.
The calculation for determining the weight of the soil on top of a rectangular and round structure is listed below:

<table>
<thead>
<tr>
<th>RECTANGULAR</th>
<th>ROUND</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>If soil is dry:</strong></td>
<td><strong>W_{SOIL} = 0.25\pi \times OD^2 \times d \times \rho_{ss}</strong> (3.3.6)</td>
</tr>
<tr>
<td>( W_{SOIL} = l \times w \times d \times \rho_d ) (3.3.3)</td>
<td></td>
</tr>
<tr>
<td><strong>If soil is submerged:</strong></td>
<td></td>
</tr>
<tr>
<td>( W_{SOIL} = l \times w \times d \times \rho_{ss} ) (3.3.5)</td>
<td></td>
</tr>
<tr>
<td><strong>W_{SOIL} = weight of earth fill (lbs.)</strong></td>
<td><strong>W_{SOIL} = weight of earth fill (lbs.)</strong></td>
</tr>
<tr>
<td><strong>l = length of the rectangular structure (ft.)</strong></td>
<td><strong>OD = outside diameter of the structure (ft.)</strong></td>
</tr>
<tr>
<td><strong>w = width of the rectangular structure (ft.)</strong></td>
<td><strong>d = depth of the structure from surface (ft.)</strong></td>
</tr>
<tr>
<td><strong>d = depth of the rectangular structure from</strong></td>
<td><strong>\rho_d = dry density of soil (lbs./ft^3)</strong></td>
</tr>
<tr>
<td><strong>\rho_d = dry density of soil (lbs./ft^3)</strong></td>
<td><strong>\rho_{ss} = simplified submerged density of soil</strong></td>
</tr>
</tbody>
</table>
| **\rho_{ss} = simplified submerged density of soil** (lbs./ft^3) | ```
3.4 Weight of Overburden Soil on Extended Base

Extending the bottom slab to create a shelf outside the walls of the underground structure adds resistance to the buoyant for FB. Additional weight is obtained from the supplementary soil above the shelf.

Determining the weight of the soils on a shelf (extended base) is the same as calculating weight of the soils on the slab. The weight of the soils on the shelf is determined by multiplying the surface area of the shelf by the depth of the shelf and the density of the submerged soil. The calculation for determining the weight of the soil on rectangular and round structures are listed below:

**RECTANGULAR**

\[
W_{\text{SOSHELF}} = 2(l + 2s + w) \times s \times d \times \rho_{ss} \quad (3.4.1)
\]

**ROUND**

\[
W_{\text{SOSHELF}} = 0.25\pi \left(OD^2 - ID^2\right) \times d \times \rho_{ss} \quad (3.4.2)
\]

where:

- \(W_{\text{SOSHELF}}\) = weight of the concrete shelf extension (lbs.)
- \(l\) = length of the rectangular structure (ft.)
- \(s\) = width of shelf (ft.)
- \(w\) = width of the rectangular structure (ft.)
- \(d\) = depth of shelf from surface (ft.)
- \(OD\) = outside diameter of shelf (ft.)
- \(ID\) = outside diameter of structure (ft.)
- \(\rho_{ss}\) = simplified submerged density of soil (lbs./ft³)

\(\rho_{ss}\) = simplified submerged density of soil (lbs./ft³)
3.5 Frictional Resistance (Extended Bases)

Adding a shelf (extended base) to the bottom of an underground structure not only adds additional vertical downward force to the structure, it may also add some frictional resistance from surrounding soil. This calculation can be obtained by adding the buoyant weight of the soil wedge created by the base extension (see figure 2). However, many engineering judgements need to be considered to determine if the frictional resistance can be obtained within the saturated conditions assumed. It is recommended for saturated soil flotation calculations the assumed soil friction angle be from 0 to 10 degrees.

![Figure 2: Frictional Resistance](image)

3.6 Bench Walls, Inverts, Etc.

If there happens to be a need for benches, inverts, etc. inside the structure, this adds weight ($W_{INVERT}$) to the structure, increasing its total vertical downward force ($TDF$). The weight of the bench, invert, etc. should be calculated and factored into the total downward vertical forces acting on the structure.

3.7 Equipment Weights (Permanent)

If there is equipment that is known to be a permanent fixture inside the structure, this adds weight ($W_{EQUIPMENT}$) to the structure, increasing its total vertical downward force ($TDF$). The weight of the equipment may be obtained by the equipment manufacturer and factored into the total downward vertical forces acting on the structure.
Section 4 – Computing Upward Buoyant Force

4.1 Buoyant Force (FB)

As stated in Archimedes’ principle, an object is buoyed up by a force equal to the weight of the fluid displaced. Mathematically the principle is defined by the equation:

\[ F_B = S_{G_w} \cdot V_d \]  

(4.1.1)

Where:

- \( F_B \) = buoyant force (lbs.)
- \( S_{G_w} \) = specific weight of water (62.4 lbs./ft\(^3\))
- \( V_d \) = total displaced volume of the fluid (ft\(^3\))

Note: Volume of structure plus the volume of any base extension applied.

When analyzing buoyancy related to precast concrete structures and applications, the application is typically below grade and stationary. Assuming the application is stationary in a fluid, analysis requires an application of the static equilibrium equation in the vertical direction, \( \Sigma F_y = 0 \) (the summation of forces in the vertical direction equal to zero).

Analyzing buoyancy related to underground structures requires an application of the same static equilibrium equation, assuming the structure to be stationary and either submerged or partially submerged in a fluid (in this case the surrounding soil/fill materials and any associated groundwater).
Section 5 – Factor of Safety

The factor of safety (FS) considers the relationship between a resisting force and a distributed force. In this case, it is the relationship between the weight of the structure, and the force of uplift caused by buoyancy.

5.2 Guide for selecting an appropriate Factor of Safety

It is recommended that the designer choose an appropriate factor of safety (FS) after reviewing information about the jobsite. The factor of safety against flotation is usually computed as the total dead weight of the structure divided by the total buoyancy uplift force.

In situations of flooding to the top of the structure and using dead weight resistance only, a FS of 1.10 is commonly used. In flood zone areas, or where high groundwater conditions exist, a FS of 1.25 should be considered. Where maximum groundwater or flood levels are not well defined, or where soil friction is included in the flotation resistance, higher FS values should be considered.

5.1 Computing the Factor of Safety (FS)

A factor of safety can be established from the following calculation:

\[
FS = \frac{\text{Total Downward Forces (TDF)}}{\text{Buoyant Force (Fb)}}
\]

\[TDF > F_b \quad \text{Structure will remain stationary}\]
\[TDF < F_b \quad \text{Structure will float or shift upward}\]

When FS is less than 1, the buoyant force will be greater than the downward forces, which means that the structure will float. When FS is greater than 1, the buoyant force will be less than the downward forces, which means that the structure will remain stationary.
Section 6 – Buoyancy Countermeasures

There are several methods that can be used in the industry to overcome a buoyancy problem. If the design of the underground structure does not meet the required factor of safety, there are ways to fix the problem. Here are a few of the different methods used to overcome a buoyancy problem, both before and after shifting or flotation.

6.1 Base Extension (Cast-in-Place or Precast)

Using the additional weight of soil by adding a shelf is a common practice used to counteract a buoyancy problem. By extending the bottom slab of the structure horizontally, this creates a shelf outside the walls of the structure and adds additional resistance to the buoyant force. The additional vertical downward force comes from the additional weight of the soil acting on the shelf ($W_{shelf}$) and the weight of the additional concrete in the shelf. The size of the shelf can be designed however large and wide is needed to resist the buoyant force.

However, limits of shipping width must be considered. In many cases, this is the most cost-effective method to counteract the buoyant force (FB). When pouring the shelf in place (cast-in-place, CIP), mechanical connections must be designed to resist the vertical shear forces. It is best to have the shelf monolithically poured with the structure if possible.

Figure 3 below illustrates the additional force that is added when a shelf is included in the design.

![Figure 3: Base Extension](image-url)
6.2 Anti-Flotation Slab
Another method used to counteract a buoyancy problem is anchoring the structure to a large concrete mass poured at the jobsite or using precast concrete transported to the site. This method can cause problems if both, are not aligned properly, then point loads may result in cracking. CIP concrete can be expensive and cause delays due to waiting for the required strength to be reached. Precast concrete alleviates such delays, but the sub-base must be sufficiently level and aligned for the two slabs to sit flush. One method of ensuring a flush fit is to use a mortar bed between the two surfaces which is recommended. (See Figure 4 below).

![Anti-Flotation Slab Diagram](image)

To design the mechanical connection between the anti-flotation slab and the structure, the net upward force must be calculated. This can be achieved by multiplying the buoyant force by the factor of safety and subtracting the total downward force.

\[
\text{Connection force (lbs.)} = FB \times FS - TDF
\]

6.3 Increase Concrete Thickness
Another method that is used to overcome buoyancy is to increase the weight of the concrete structure. This can be accomplished by increasing concrete thickness (i.e. walls and slabs). Increasing the thickness of the walls and slabs can add significantly to the total downward force but this may not always be the most economical solution. This can be a costly alternative due to the increasing material and production costs.

6.4 Install Structure Deeper and Fill Base with Additional Concrete
An additional method used to overcome buoyancy is to install the precast structure deeper than required for its functional purposes and fill the added depth with additional concrete. This will add additional soil weight on top of the structure to oppose buoyant forces. Also, with the structure being deeper in the soil, some contractors opt to pour additional concrete into the base of the installed precast structure. This will again add more weight to the structure, which helps overcome buoyance \((TDF > F_B)\).
6.5 Anchor the Structure

Another method used to overcome buoyancy is to provide anchors to help tie down the structure. These anchors are typically independent of the structure as illustrated in the figure below. The designer must analyze the stresses and strains in the proposed hold down strapping materials are appropriate and that the corrosion resistance of the material will meet or exceed the design life for the buried structure itself.

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Figure 5: Anchor Structure

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Section 7 – Recap

To review all the other sections above and determine a plan of action, these are the essential considerations that must be made when writing specifications about buoyant forces.

1. Use data obtained from either a soils report or other means described in the first part of Section II. Then from the information obtained, determine an accurate water level in the ground (WL) and a reasonable factor of safety (FS), which are suitable for the given jobsite conditions. Try to produce an optimally designed structure, since designing an underground structure with a higher than needed WL and FS will result in higher production cost as well as other underlying costs.

2. Selection of appropriate backfill material. This is needed when backfilling over and around the structure as well as the shelf as noted in section 3.5.

3. When the possibility for flotation exists, the precast concrete designer must develop a method to resist buoyancy and back the method with calculations. In those calculations, the designer must show how the structure will resist the buoyant force and determine a FS. Developing a plan such as this will help the precaster provide the most cost-effective product.

4. Submit calculations to the appropriate owner or authority of the precast product for approval. These calculations need to include the depth of the groundwater level below grade and state the FS and supply data and information to why the FS is such. Factors such as the weight of the structure, the weight of the soil over the structure and shelves, to name a few, should be included.
It is essential when designing an underground precast structure that the contractor and the precast manufacturer work together. The precast manufacturer needs vital information which the contractor usually has access to, and this information will help the precast manufacturer design the most cost-effective structure which prevents floatation or shifting. The most cost-effective method is determined after considering the site conditions, manufacturing methods, and the contractor handling and installation issues.

**Section 8 – Examples**

**8.1 Four examples illustrating how the various countermeasures can be applied are as follows:**

Problem: A structure is to be installed underground in a location where the water table is at grade. Verify that the following structure will resist buoyancy using a factor of safety of 1.1. (See Figure 6 below).

<table>
<thead>
<tr>
<th>Inside Length (lL)</th>
<th>Top Slab Thickness (TS)</th>
<th>Outside Length (OL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 ft.</td>
<td>8 in. (0.67 ft.)</td>
<td>11.33 ft.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inside Width (IW)</th>
<th>Wall Thickness (WT)</th>
<th>Outside Width (OW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 ft.</td>
<td>8 in. (0.67 ft.)</td>
<td>9.33 ft.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Inside Height (IH)</th>
<th>Bottom Slab Thickness (BS)</th>
<th>Outside Height (OH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 ft.</td>
<td>8 in. (0.67 ft.)</td>
<td>13.33 ft.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Depth of Earth Fill (f)</th>
<th>Water Table Depth (d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ft.</td>
<td>0 ft. at grade</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density (Unit Weight) Concrete (ρC)</th>
<th>Top Slab Opening (OTS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>150 lbs./ft³</td>
<td>24 in. diameter (2 ft.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density (Unit Weight) Soil (ρS)</th>
<th>Wall Openings (WO)</th>
</tr>
</thead>
<tbody>
<tr>
<td>120 lbs./ft³</td>
<td>36 in. diameter (3 ft.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Density (Unit Weight) Water (ρW)</th>
<th>Safety Factor Required (FS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>62.4 lbs./ft³</td>
<td>1.1</td>
</tr>
</tbody>
</table>

Simplified density of submerged soil (ρSS) = 57.6 lbs/ft³
Step 1: Calculate all downward forces

Concrete Weight: \( W_{	ext{CON}} = (O_L \times O_W \times O_H - l_L \times l_W \times l_H) \times \rho_c \) \hspace{1cm} (8.1.1)

\[
W_{	ext{CON}} = (11.33 \text{ ft} \times 9.33 \text{ ft} \times 13.33 \text{ ft} - 10 \text{ ft} \times 8 \text{ ft} \times 12 \text{ ft}) \times 150 \text{ lbs./ft}^3
\]
\[
W_{	ext{CON}} = 67,364.95 \text{ lbs.}
\]

Weight of Earth Fill: \( W_{	ext{FILL}} = O_L \times O_W \times f \times \rho_{SS} \) \hspace{1cm} (8.1.2)

\[
W_{	ext{FILL}} = 11.33 \text{ ft} \times 9.33 \text{ ft} \times 1 \text{ ft} \times 57.6 \text{ lbs./ft}^3
\]
\[
W_{	ext{FILL}} = 6,088.83 \text{ lbs.}
\]

Top Slab Opening and Soil Weight over Opening (subtracted quantity):

\( (T_{	ext{OSW}}) = (\pi \times (O_{TS}/2)^2 \times T_s) \times \rho_c + (\pi \times (O_{TS}/2)^2 \times f) \times \rho_{SS} \) \hspace{1cm} (8.1.3)

\[
T_{	ext{OSW}} = (\pi \times (2 \text{ ft.}/2)^2 \times 0.67 \text{ ft.}) \times 150 \text{ lbs./ft}^3 + (\pi \times (2 \text{ ft.}/2)^2 \times 1.0 \text{ ft.}) \times 57.6 \text{ lbs./ft}^3
\]
\[
T_{	ext{OSW}} = 496.69 \text{ lbs}
\]

Wall Openings (subtracted quantity):

\( (W_O) = [(\pi \times (W/T)^2 \times W_T) \times \rho_c] \times 2 \) \hspace{1cm} (8.1.4)

\[
W_O = [(\pi \times (3 \text{ ft.}/2)^2 \times 0.67 \text{ ft.}) \times 150 \text{ lbs./ft}^3] \times 2
\]
\[
W_O = 1420.79 \text{ lbs}
\]

Total Downward Force:

\( (TDF) = W_{	ext{CON}} + W_{	ext{FILL}} - T_{	ext{OSW}} - W_O \) \hspace{1cm} (8.1.5)

\[
TDF = 67,364.95 \text{ lbs.} + 6,088.83 \text{ lbs.} - 496.69 \text{ lbs.} - 1420.79 \text{ lbs.}
\]
\[
TDF = 71,536.30 \text{ lbs}
\]
Step 2: Calculate upward buoyant force

\[
(F_B) = \rho_w \times V_d
\]

where:

\[
\rho_w = 62.4 \text{ lbs./ft}^3, \text{ and}
\]

\[
V_d = O_L \times O_W \times O_H
\]

\[
F_B = 62.4 \text{ lbs./ft}^3 \times [11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 13.33 \text{ ft}]
\]

\[
F_B = 87,927.82 \text{ lbs}
\]

Step 3: Calculate the difference between total downward force and upward buoyant force

\[
\Delta = TDF - F_B
\]

\[
\Delta = 71,536.30 \text{ lbs.} - 87,927.82 \text{ lbs.}
\]

\[
\Delta = -16,391.52 \text{ lbs. (upward)}
\]

Step 4: Calculate the safety factor

\[
(FS) = \frac{TDF}{F_B}
\]

\[
FS = \frac{71,536.30 \text{ lbs.}}{87,927.82 \text{ lbs.}}
\]

\[
FS = 0.81
\]

Since 0.81 < 1.10 the structure will not resist buoyancy forces with the desired FS of 1.1

Following are six optional solutions for consideration:

1. Make the inside of the structure deeper and add excess concrete fill to the inside of the structure.
2. Add weight by increasing the wall and slab concrete thicknesses.
3. A. Add an extension to the outside of the base to engage the soil above the extension,
   or
   B. Increase the shelf extension from 6 inches to 8 inches.
4. Obtain additional soil data to determine actual submerged soil density \( \rho_s \) and recalculate buoyancy factors.
5. If an extension to the outside of the base slab is chosen you could also assume a soil wedge will develop adding to the downward force to resist flotation.
6. Add a separate anti-flotation slab below the base slab.
OPTION #1: Make the structure deeper and add concrete fill to the inside.

Step 1: Calculate the required additional concrete weight

\[(W_{\text{ADD}}) = (F_b \times FS) - TDF \quad (8.1.10)\]

\[W_{\text{ADD}} = (87,927.82 \text{ lbs.} \times 1.1) - 71,536.30 \text{ lbs.}\]

\[W_{\text{ADD}} = 25,184.30 \text{ lbs.}\]

Step 2: Calculate the additional structure depth required

\[(D_{\text{ADD}}) = \frac{V_{\text{ADD}}}{A_{\text{OUTSIDE}}} \quad (8.1.11)\]

where:

\[V_{\text{ADD}} = \frac{W_{\text{ADD}}}{(\rho_c - \rho_w)} \quad (8.1.12)\]

\[A_{\text{OUTSIDE}} = (O_l \times O_w) \quad (8.1.13)\]

\[D_{\text{ADD}} = \frac{25,184.30 \text{ lbs.}}{150 \text{ lbs./ft.}^3 - 62.4 \text{ lbs./ft.}^3} \quad (8.1.14)\]

\[D_{\text{ADD}} = \frac{(11.33 \text{ ft.} \times 9.33 \text{ ft.})}{(11.33 \text{ ft.} \times 9.33 \text{ ft.})}\]

\[D_{\text{ADD}} = 287.49 \text{ ft.} / 105.71 \text{ ft.}\]

\[D_{\text{ADD}} = 2.72 \text{ ft. (round up to 3.0 ft.)}\]

Step 3: Revise given information inside and outside height (see given below and Figure 7)

Inside Length \((l_i) = 10 \text{ ft.} \) Top Slab Thickness \((T_s) = 8 \text{ in.} \) Outside Length \((O_L) = 11.33 \text{ ft.}\)

Inside Width \((l_w) = 8 \text{ ft.} \) Wall Thickness \((W_t) = 8 \text{ in.}\) Outside Width \((O_W) = 9.33 \text{ ft.}\)

Inside Height \((l_H) = 15 \text{ ft.} \) Bottom Slab Thickness \((B_s) = 8 \text{ in.}\) Outside Height \((O_H) = 16.33 \text{ ft.}\)

Depth of Earth Fill \((f) = 1 \text{ ft.} \) Water Table Depth \((d) = 0 \text{ ft. at grade}\)

Density (Unit Weight) Concrete \((\rho_c) = 150 \text{ lbs./ft}^3\) Top Slab Opening \((O_{ts}) = 24 \text{ in. diameter (2 ft.)}\)

Density (Unit Weight) Soil \((\rho_s) = 120 \text{ lbs./ft}^3\) Wall Openings \((W_o) = 36 \text{ in. diameter (3 ft.) 2 each}\)

Density (Unit Weight) Water \((\rho_w) = 62.4 \text{ lbs./ft}^3\) Safety Factor Required \((FS) = 1.1\)

Simplified density of submerged soil \((\rho_{sp}) = 57.6 \text{ lbs/ft}^3\)
Step 4: Recalculate all of the downward forces

Concrete Weight: \( W_{\text{CON}} = (O_L \times O_W \times O_H - I_L \times I_W \times I_H) \times \rho_C \)  
\[ (8.1.1) \]
\[ W_{\text{CON}} = (11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 16.33 \text{ ft.} - 10 \text{ ft.} \times 8 \text{ ft.} \times 15 \text{ ft.}) \times 150 \text{ lbs./ft}^3 \]
\[ W_{\text{CON}} = 78,933.95 \text{ lbs.} \]

Weight of Earth Fill: \( W_{\text{FILL}} = O_L \times O_W \times f \times \rho_{SS} \)  
\[ (8.1.2) \]
\[ W_{\text{FILL}} = 11.33 \text{ ft} \times 9.33 \text{ ft} \times 1 \text{ ft} \times 57.6 \text{ lbs./ft}^3 \]
\[ W_{\text{FILL}} = 6,088.83 \text{ lbs.} \]

Weight of Infill: \( W_{\text{INFILL}} = I_L \times I_W \times DADD \times \rho_C \)  
\[ (8.1.15) \]
\[ W_{\text{INFILL}} = 10 \text{ ft} \times 8 \text{ ft} \times 3 \text{ ft} \times 150 \text{ lbs./ft}^3 \]
\[ W_{\text{INFILL}} = 36,000 \text{ lbs.} \]

Top Slab Opening and Soil Weight over Opening (subtracted quantity):
\( (T_{\text{SOsw}}) = (\pi \times (O_{TS}/2)^2 \times T_{S}) \times \rho_C + (\pi \times (O_{TS}/2)^2 \times f) \times \rho_{SS} \)  
\[ (8.1.3) \]
\[ T_{\text{SOsw}} = (\pi \times (2 \text{ ft.}/2)^2 \times 0.67 \text{ ft.}) \times 150 \text{ lbs./ft}^3 + (\pi \times (2 \text{ ft.}/2)^2 \times 1.0 \text{ ft.}) \times 57.6 \text{ lbs./ft}^3 \]
\[ T_{\text{SOsw}} = 496.69 \text{ lbs.} \]
Wall Openings (subtracted quantity):
\[
(W_O) = \left( \pi \times \left( \frac{W_O}{2} \right)^2 \times W_T \right) \times \rho_c \times 2
\] (8.1.4)
\[
W_O = \left( \frac{\pi \times (3 \text{ ft./2})^2 \times 0.67 \text{ ft.}}{150 \text{ lbs./ft.}^3} \right) \times 2
\]
\[
W_O = 1420.79 \text{ lbs}
\]

Total Downward Force:
\[
(TDF) = W_{CON} + W_{FILL} + W_{INFILL} - T_{SOSW} - W_O
\] (8.1.5)
\[
TDF = 78,933.95 \text{ lbs.} + 6,088.83 \text{ lbs.} + 36,000 \text{ lbs.} - 496.69 \text{ lbs.} - 1420.79 \text{ lbs.}
\]
\[
TDF = 119,105.30 \text{ lbs.}
\]

Step 5: Calculate upward buoyant force
\[
(F_B) = \rho_w \times V_d
\] (8.1.6)
where:
\[
\rho_w = 62.4 \text{ lbs./ft}^3, \text{ and}
\]
\[
V_d = O_L \times O_W \times (O_H)
\] (8.1.7)
\[
F_B = 62.4 \text{ lbs./ft}^3 \times [11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 16.33 \text{ ft}]
\]
\[
F_B = 107,716.52 \text{ lbs}
\]

Step 6: Calculate the difference between total downward force and upward buoyant force
\[
\Delta = TDF - F_B
\] (8.1.8)
\[
\Delta = 119,105.30 \text{ lbs.} - 107,716.52 \text{ lbs.}
\]
\[
\Delta = 11,388.78 \text{ lbs. (downward)}
\]

Step 7: Calculate Safety Factor
\[
(FS) = \frac{TDF}{F_B}
\] (8.1.9)
\[
FS = \frac{119,105.30 \text{ lbs.}}{107,716.52 \text{ lbs.}}
\]
\[
FS = 1.11
\]

The structure will resist buoyancy forces with the desired FS of 1.1.
OPTION #2: Increase concrete wall, and slab thickness.

Using the same interior length, width, and height, let’s assume a 12” (1 ft.) top slab, 14” (1.08 ft.) walls, and 12” (1 ft.) bottom slab (see given below and Figure 8).

- Inside Length \( l_I \) = 10 ft.  
  - Top Slab Thickness \( T_S \) = 12 in. (1.0 ft.)  
  - Outside Length \( O_L \) = 12.17 ft.  

- Inside Width \( l_W \) = 8 ft.  
  - Wall Thickness \( W_T \) = 13 in. (1.08 ft.)  
  - Outside Width \( O_W \) = 10.17 ft.  

- Inside Height \( l_H \) = 15 ft.  
  - Bottom Slab Thickness \( B_S \) = 12 in. (1 ft.)  
  - Outside Height \( O_H \) = 14.0 ft  

Depth of Earth Fill \( f \) = 1 ft.  
Water Table Depth \( d \) = 0 ft. at grade

- Density (Unit Weight) Concrete \( \rho_C \) = 150 lbs./ft\(^3\)  
  - Top Slab Opening \( O_T S \) = 24 in. diameter (2 ft.)  

- Density (Unit Weight) Soil \( \rho_S \) = 120 lbs./ft\(^3\)  
  - Wall Openings \( W_O \) = 36 in. diameter (3 ft.) 2 each  

- Density (Unit Weight) Water \( \rho_W \) = 62.4 lbs./ft\(^3\)  
  - Safety Factor Required \( F_S \) = 1.1  

Simplified density of submerged soil \( \rho_{SS} \) = 57.6 lbs/ft\(^3\)

Figure 8
Step 1: Calculate all of the downward forces

Concrete Weight: \( W_{CON} = (O_L \times O_W \times O_H - I_L \times I_W \times I_H) \times \rho_c \) \hspace{1cm} (8.1.1)
\[ W_{CON} = (12.33 \text{ ft.} \times 10.17 \text{ ft.} \times 14 \text{ ft.} - 10 \text{ ft.} \times 8 \text{ ft.} \times 12 \text{ ft.}) \times 150 \text{ lbs./ft}^3 \]
\[ W_{CON} = 119,331.75 \text{ lbs.} \]

Weight of Earth Fill: \( W_{FILL} = O_L \times O_W \times f \times \rho_{SS} \) \hspace{1cm} (8.1.2)
\[ W_{FILL} = 12.17 \text{ ft} \times 10.33 \text{ ft} \times 1 \text{ ft} \times 57.6 \text{ lbs./ft}^3 \]
\[ W_{FILL} = 7,421.23 \text{ lbs.} \]

Top Slab Opening and Soil Weight over Opening (subtracted quantity):
\( W_{SO} = (\pi \times (O_{TS}/2)^2 \times Ts) \times \rho_c + (\pi \times (O_{TS}/2)^2 \times f) \times \rho_{SS} \) \hspace{1cm} (8.1.3)
\[ W_{SO} = (\pi \times (2 \text{ ft.}/2)^2 \times 1 \text{ ft.}) \times 150 \text{ lbs./ft}^3 + (\pi \times (2 \text{ ft.}/2)^2 \times 1.0 \text{ ft.}) \times 57.6 \text{ lbs./ft}^3 \]
\[ W_{SO} = 652.19 \text{ lbs.} \]

Wall Openings (subtracted quantity):
\( W_{OW} = [\pi \times (O_{WO}/2)^2 \times W_{T}] \times \rho_c \times 2 \text{ openings} \) \hspace{1cm} (8.1.4)
\[ W_{OW} = [\pi \times (3 \text{ ft.}/2)^2 \times 1.08 \text{ ft.}) \times 150 \text{ lbs./ft}^3] \times 2 \]
\[ W_{OW} = 2,290.22 \text{ lbs.} \]

Total Downward Force:
\[ TDF = W_{CON} + W_{FILL} - W_{SO} - W_{OW} \] \hspace{1cm} (8.1.5)
\[ TDF = 119,331.75 \text{ lbs.} + 7,421.23 \text{ lbs.} - 652.19 \text{ lbs.} - 2290.22 \text{ lbs.} \]
\[ TDF = 123,810.57 \text{ lbs.} \]

Step 2: Calculate upward buoyant force
\( F_B = \rho_w \times V_d \) \hspace{1cm} (8.1.6)
where:
\[ \rho_w = 62.4 \text{ lbs./ft}^3, \text{ and} \]
\[ V_d = O_L \times O_W \times O_H \] \hspace{1cm} (8.1.7)
\[ F_B = 62.4 \text{ lbs./ft}^3 \times [12.17 \text{ ft.} \times 10.33 \text{ ft.} \times 14 \text{ ft}] \]
\[ F_B = 109,825.59 \text{ lbs.} \]

Step 3: Calculate the difference between total downward force and upward buoyant force
\[ \Delta = TDF - F_B \] \hspace{1cm} (8.1.8)
\[ \Delta = 123,810.57 \text{ lbs.} - 109,825.59 \text{ lbs.} \]
\[ \Delta = 13.984.98 \text{ lbs. (downward)} \]

Step 4: Calculate Safety Factor
\( FS = \frac{TDF}{F_B} \) \hspace{1cm} (8.1.9)
\[ FS = 123,810.57\text{lbs.} / 109825.59 \text{ lbs.} \]
\[ FS = 1.13 \]
The structure will resist buoyancy forces with the desired FS greater than 1.1
**OPTION #3a: Add an extension to the outside of the base slab to engage the soil outside the structure.**

The base extension is the same thickness as the bottom slab of the structure and extends 6 in. (0.5 ft.) from the existing side walls on all sides.

- Inside Length ($l_L$) = 10 ft.
- Top Slab Thickness ($T_s$) = 8 in. (0.67 ft.)
- Outside Length ($O_L$) = 11.33 ft.

- Inside Width ($I_W$) = 8 ft.
- Wall Thickness ($W_T$) = 8 in. (0.67 ft.)
- Outside Width ($O_W$) = 9.33 ft.

- Inside Height ($l_H$) = 12 ft.
- Bottom Slab Thickness ($B_s$) = 8 in. (0.67 ft.)
- Outside Height ($O_H$) = 13.33 ft.

- Depth of Earth Fill ($f$) = 1 ft.
- Water Table Depth ($d$) = 0 ft. at grade
- Base Extension ($s$) = 6 in. (0.5 ft.)

- Density (Unit Weight) Concrete ($\rho_C$) = 150 lbs./ft$^3$
- Top Slab Opening ($O_{TS}$) = 24 in. diameter (2 ft.)

- Density (Unit Weight) Soil ($\rho_S$) = 120 lbs./ft$^3$
- Wall Openings ($W_O$) = 36 in. diameter (3 ft.)

- Density (Unit Weight) Water ($\rho_W$) = 62.4 lbs./ft$^3$
- Safety Factor Required ($FS$) = 1.1

Simplified density of submerged soil ($\rho_{SS}$) = 57.6 lbs/ft$^3$

![Figure 9](image-url)
Step 1: Calculate all downward forces

Concrete Weight: \(W_{\text{CON}} = (O_L \times O_W \times O_H - I_L \times I_W \times I_H) \times p_C\)  (8.1.1)

\[
W_{\text{CON}} = (11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 13.33 \text{ ft.} - 10 \text{ ft.} \times 8 \text{ ft.} \times 12 \text{ ft.}) \times 150 \text{ lbs./ft}^3
\]

\[
W_{\text{CON}} = 67,364.95 \text{ lbs.}
\]

Concrete Weight of Shelf Extension:

\[
(W_{\text{SHELF}}) = 2 \times (O_L + 2s + O_W) \times B_s \times p_C \times s\]  (3.1.5)

\[
W_{\text{SHELF}} = 2 \times [11.33 \text{ ft.} + (2 \times 0.5 \text{ ft.}) + 9.33 \text{ ft.}] \times 0.67 \text{ ft.} \times 150 \text{ lbs./ft}^3 \times 0.5 \text{ ft.}
\]

\[
W_{\text{SHELF}} = 2,176.83 \text{ lbs.}
\]

Weight of Earth Fill: \(W_{\text{FILL}} = O_L \times O_W \times f \times p_{SS}\)  (8.1.2)

\[
W_{\text{FILL}} = 11.33 \text{ ft} \times 9.33 \text{ ft} \times 1 \text{ ft} \times 57.6 \text{ lbs./ft}^3
\]

\[
W_{\text{FILL}} = 6,088.83 \text{ lbs.}
\]

Buoyant Weight of Soil Engaged by Shelf Extension:

\[
(BWSE) = [(\text{Perimeter of structure} \times s) \times \text{soil height} \times p_{SS}]
\]

where:

\[
BWSE = [2 \times (O_L + s) + 2 \times (O_W + s)] \times s * (I_H + T_S + f) * p_{SS}\]  (8.1.16)

\[
BWSE = [2 \times (11.33 \text{ ft.} + 0.5 \text{ ft.}) + 2 \times (9.33 \text{ ft.} + 0.5 \text{ ft.})] \times 0.5 \text{ ft.} \times (12 \text{ ft.} + 0.67 \text{ ft.} + 1 \text{ ft.}) \times (57.6 \text{ lbs./ft}^3)
\]

\[
BWSE = 17,054.91 \text{ lbs.}
\]

Top Slab Opening and Soil Weight over Opening (subtracted quantity):

\[
(TS_{\text{OSW}}) = (\pi \times (OTS/2)^2 \times TS) \times p_C + (\pi \times (OTS/2)^2 \times f) \times p_{SS}\]  (8.1.3)

\[
TS_{\text{OSW}} = (\pi \times (2 \text{ ft./2})^2 \times 0.67 \text{ ft.}) \times 150 \text{ lbs./ft}^3 + (\pi \times (2 \text{ ft./2})^2 \times 1.0 \text{ ft.}) \times 57.6 \text{ lbs./ft}^3
\]

\[
TS_{\text{OSW}} = 496.69 \text{ lbs.}
\]

Wall Openings (subtracted quantity):

\[
(WO \ \text{WEIGHT}) = [(\pi \times (Wo/2)^2 \times WT) \times p_w] \times 2 \text{ openings}\]  (8.1.4)

\[
WO \ \text{WEIGHT} = [(\pi \times (3 \text{ ft./2})^2 \times 0.67 \text{ ft.}) \times 150 \text{ lbs./ft}^3] \times 2
\]

\[
WO \ \text{WEIGHT} = 1,420.79 \text{ lbs.}
\]

Total Downward Force with Shelf

\[
(TDF) = W_{\text{CON}} + W_{\text{SHELF}} + W_{\text{FILL}} + BWSE - TS_{\text{OSW}} - W_O\]  (8.1.5)

\[
TDF = 67,364.95 \text{ lbs.} + 2,176.83 \text{ lbs.} + 6,088.83 \text{ lbs.} - 496.69 \text{ lbs.} - 1,420.79 \text{ lbs.}
\]

\[
TDF = 107,822.95 \text{ lbs.}
\]

Step 2: Calculate upward buoyant force

\[
(F_B) = \rho_w \times V_d\]  (8.1.6)

where:

\[
\rho_w = 62.4 \text{ lbs./ft}^3, \text{ and}
\]

\[
V_d = [O_L \times O_W \times O_H] + [2 \times (O_L + 2s + O_W) \times T_S \times s]\]

\[
F_B = 62.4 \text{ lbs./ft}^3 \times [11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 13.33 \text{ ft.}] + [2 \times (11.33 \text{ ft.} + 1 \text{ ft.} + 9.33 \text{ ft.}) \times 0.67 \text{ ft.} \times 0.5 \text{ ft.}]
\]

\[
F_B = 88,833.38 \text{ lbs.}
\]
Step 3: Calculate the difference between total downward force and upward buoyant force.

\[ \Delta = \text{TDF} - F_b \]  
(8.1.8)
\[ \Delta = 107,822.95 \text{ lbs.} - 88,833.83 \text{ lbs.} \]
\[ \Delta = 18,989.12 \text{ lbs. (downward)} \]

Step 4: Calculate Safety Factor

\[ (FS) = \frac{\text{TDF}}{F_b} \]  
(8.1.9)
\[ FS = \frac{107,822.95 \text{ lbs.}}{88,833.38 \text{ lbs.}} \]
\[ FS = 1.21 \]

The required FS is 1.1, the resulting FS of 1.21 is adequate.

**OPTION #4: Obtain additional backfill soil geotechnical data and use actual submerged soil density**

Obtain additional backfill soil geotechnical data that provides specific gravity and dry soil density information to calculate the actual submerged soil density, \( \rho_s \), described in Section 3.3.

Recalculate the downward force replacing \( \rho_s \) with this \( \rho_s \) value. With a soil specific gravity provided of 2.68 and dry density of 118 lbs./ft\(^3\) the submerged soil density is:

\[ \rho_s = [1 - (1/G_s)] \times \rho_d \]  
(3.3.1)
\[ \rho_s = [1 - (1/2.68)] \times 118 \text{ lbs./ft}^3 \]
\[ \rho_s = 73.97 \text{ lbs./ft}^3 \]

By using this soil density and performing the same calculations we did under option 3, we get:

\( W_{CON} = 67,364.95 \text{ lbs.} \)
\( W_{SHELF} = 2,176.83 \text{ lbs.} \)
\( W_{FILL} = 7,819.29 \text{ lbs.} \)
\( BWSE = 21,901.94 \text{ lbs.} \)
\( TS_{osw} = -548.11 \text{ lbs.} \)
\( W_{o \text{ weight}} = -1420.79 \text{ lbs} \)
\( \text{TDF} = 97,294.11 \text{ lbs} \)
F_b = 88,833.38 lbs

\( \Delta = 8,460.73 \text{ lbs.} \)

FS = 1.1

The required FS is 1.1, so using actual soil density works with a 6” base extension.

---

**OPTION #5: Calculate additional downward force due to soil wedge**

Since the factor of safety without considering a soil wedge is slightly greater than 1.0, compute the weight of soil within the soil wedge. Assume a fiction angle of \( \alpha = 10^\circ \) (conservative)

![Figure 10](image)

Through basic trigonometry, we need the top measurement \( x \) of the wedge. (Figure 11)

The height of the wedge goes from the top of the shelf to the top of the soil

\[ = l_s + T_s + f = 12 \text{ ft.} + 0.67 \text{ ft.} + 1 \text{ ft.} = 13.67 \text{ ft.} \]

We know through trigonometric equations with right angle triangles that \( \tan 10^\circ = \frac{x}{13.67 \text{ ft.}} \)

So \( x = \tan 10^\circ \times 13.67 \text{ ft.} \approx 2.41 \text{ ft.} \)
Knowing this dimension now enables us to calculate the weight of the soil wedge. We are calculating only the wedge portion around the outer perimeter of the extended base (Figure 12).
Volume of a hollow obelisk such as this is (Figure 13)

\[
\text{Volume} = \left[ h/6 \left( AB + aB + 2(ab + AB) \right) - abh \right]
\]

In our case

\[
\begin{align*}
A &= 12.33 + 2.41 + 2.41 = 17.15 \text{ ft.} \\
B &= 10.33 + 2.41 + 2.41 = 15.15 \text{ ft.} \\
H &= 13.67 \text{ ft.}
\end{align*}
\]

Volume of soil wedge = 
\[
\frac{13.67}{6} \times \left( (15.15 \times 10.33) + (12.33 \times 17.15) \\
+ 2 \times (10.33 \times 12.33) + (17.15 \times 15.15) \right) - (10.33 \times 12.33 \times 13.67)
\]

Volume of soil wedge = 852.39 ft\(^3\)

Weight of the soil wedge = Volume of soil wedge \times \text{(density submerged soil } \rho_{ss} \text{ )}

\[
= 852.39 \text{ ft}^3 \times (57.6 \text{ lbs/ft}^3)
\]

= 49,097.66 lbs

Calculate new TDF = 90,768.04 lbs. + 49,097.66 lbs. = 139,865.70 lbs

Calculate new FS

\[
(\text{FS}) = \frac{\text{TDF}}{F_b} \quad (8.1.9)
\]

FS = 139,865.7 lbs. / 88,833.83 lbs.

FS = 1.57

The resulting FS of 1.57 satisfies the requirements.
OPTION #6: Add a separate anti-flotation slab below the base slab of the original structure.

Using the original structure geometry from option 3 without the soil wedge with a FS = 1.02, add a separate anti-flotation slab below the existing base slab of the structure. Assume the separate anti-flotation slab to have a thickness of 1 ft. and the same outside dimensions as the base slab and extension from the structure in option 3.

Inside Length ($l_l$) = 10 ft.  Top Slab Thickness ($T_s$) = 8 in. (0.67 ft.)  Outside Length ($O_l$) = 11.33 ft.

Inside Width ($l_w$) = 8 ft.  Wall Thickness ($W_r$) = 8 in. (0.67 ft.)  Outside Width ($O_w$) = 9.33 ft.

Inside Height ($l_h$) = 12 ft.  Bottom Slab Thickness ($B_s$) = 8 in. (0.67 ft.)  Outside Height ($O_h$) = 13.33 ft

Depth of Earth Fill ($f$) = 1 ft.  Water Table Depth ($d$) = 0 ft. at grade  Base Extension ($s$) = 6 in (0.5 ft.)

Anti-Flotation Slab Thickness (AFT) = 1 ft  Anti-Flotation Slab Protrusion all sides (AFP) = 6” (0.5 ft.)

Density (Unit Weight) Concrete ($\rho_c$) = 150 lbs./ft$^3$  Top Slab Opening (OTS) = 24 in. diameter (2 ft.)

Density (Unit Weight) Soil ($\rho_s$) = 120 lbs./ft$^3$  Wall Openings (Wo) = 36 in. diameter (3 ft.) 2 each

Density (Unit Weight) Water ($\rho_w$) = 62.4 lbs./ft$^3$  Safety Factor Required (FS) = 1.1

Simplified density of submerged soil ($\rho_{ss}$) = 57.6 lbs/ft$^3$

Step 1: Calculate all downward forces

Concrete Weight: ($W_{CON}$) = ($O_l$ x $O_w$ x $O_h$ – $l_l$ x $l_w$ x $l_h$) x $\rho_c$  

\[
W_{CON} = (11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 13.33 \text{ ft.} - 10 \text{ ft.} \times 8 \text{ ft.} \times 12 \text{ ft.}) \times 150 \text{ lbs./ft}^3
\]

\[
W_{CON} = 67,364.95 \text{ lbs.}
\]

Concrete Weight of Shelf Extension:

\[
(W_{SHELF}) = 2 \times (O_l + 2_s + O_w) \times B_s \times \rho_c \times s
\]

\[
W_{SHELF} = 2 \times [11.33 \text{ ft.} + (2 \times 0.5 \text{ ft.}) + 9.33 \text{ ft.}] \times 0.67 \text{ ft.} \times 150 \text{ lbs./ft}^3 \times 0.5 \text{ ft.}
\]

\[
W_{SHELF} = 2,176.83 \text{ lbs.}
\]

Weight of Earth Fill:

\[
(W_{FILL}) = O_l \times O_w \times f \times \rho_{ss}
\]

\[
W_{FILL} = 11.33 \text{ ft.} \times 9.33 \text{ ft.} \times 1 \text{ ft.} \times 57.6 \text{ lbs./ft}^3
\]

\[
W_{FILL} = 6,088.83 \text{ lbs}
\]
Buoyant Weight of Soil Engaged by Shelf Extension:
(BWSE) = [(Perimeter of structure x s) x soil height x \( \rho_{SS} \)]
where:
Perimeter of structure = 2 x (Ol + s) + 2 x (Ow + s) = 2 x (11.33 ft. + 0.5 ft) + 2 x (9.33 ft. + 0.5 ft)
Perimeter of structure = 43.32 ft.
Soil height = Ih + Ts + f = 12 ft. + 0.67 ft. + 1 ft. = 13.67 ft.
BWSE = 43.32 ft. x 0.5 ft. x 13.67 ft. x 57.6 lbs/ft
BWSE = 17,054.91 lbs.

Top Slab Opening and Soil Weight over Opening (subtracted quantity):
(TSosw) = (\( \pi \) x (ots/2)² x Ts) x \( \rho_c \) + (\( \pi \) x (ots/2)² x f) x \( \rho_{SS} \))
TSosw = (\( \pi \) x (2 ft./2)² x 0.67 ft.) x 150 lbs./ft³ + (\( \pi \) x (2 ft./2)² x 1.0 ft.) x 57.6 lbs./ft³
TSosw = 496.69 lbs

Wall Openings (subtracted quantity):
(Wo weight) = [(\( \pi \) x (ow/2)² x wt) x \( \rho_d \)] x 2
Wo weight = [(\( \pi \) x (3 ft./2)² x 0.67 ft.) x 150 lbs./ft³] x 2
Wo weight = 1420.79 lbs.

Total Downward Force with Shelf:
(TDF) = WCON + BWSE + Wfill - TSosw - Wo
(TDF) = 67,364.95 lbs. + 17,054.91 lbs + 6,088.83 lbs. - 496.69 lbs. - 1420.79 lbs.
TDF = 90,768.04 lbs

Step 2: Calculate upward buoyant force
(FB) = \( \rho_w \) x Vd
where: \( \rho_w \) = 62.4 lbs/ft³ and
Vd = (Ol x Ow x (Ow + f)) + [2 x (Ol + 2s + Ow) x Ts x s]
Fb = 62.4 lbs./ft³ x (11.33 ft. x 9.33 ft. x 9.33 ft.) + [2 x (11.33 ft. + 1 ft. + 9.33 ft.) x 0.67 ft. x 0.5 ft.]
FB = 88,833.83 lbs.

Step 3: Calculate the difference between total downward force and upward buoyant force
\( \Delta \) = TDF - FB
\( \Delta \) = 90,768.04 lbs. - 88,833.83 lbs.
\( \Delta \) = 1,934.21 lbs (downward)

Step 4: Calculate Safety Factor
(FS) = TDF / FB
FS = 90,768.04 lbs / 88,833.83 lbs.
FS = 1.02
The required FS is 1.1, the resulting FS = 1.02 does not work.
Step 5: Calculate the additional downward force created by the slab
Slab volume: 12.33 ft. x 10.33 ft. x 1 ft. = 127.37 ft$^3$
Slab weight: 127.37 ft$^3$ x (150 lbs./ft$^3$) = 19,105.34 lbs
Add slab weight to total downward force from above:
TDF = 90,768.04 lbs. + 19,105.34 lbs.
TDF = 109,873.38 lbs

Step 6: Calculate new buoyant force
(FBSLAB) = $\rho_w *$ Slab volume = 62.4 lbs/ft$^3$ x 127.37 ft$^3$ = 7,947.89 ft$^3$
Fb = 88,833.83 lbs + 7,947.89 lbs = 96,781.72 lbs

Step 7: Calculate new FS
FS = 109,873.38 lbs / 96,781.72 lbs
FS = 1.14 > 1.1 OK

Could we potentially reduce the anti-flotation slab thickness to 10 in.?
Slab volume: 12.33 ft. x 10.33 ft. x 0.83 ft. = 105.72 ft$^3$
Slab weight: 105.72 ft$^3$ x 150 lbs./ft$^3$ = 15,857.43 lbs
Add slab weight to total downward force from above:
TDF = 90,768.04 lbs. + 15,857.43 lbs.
TDF = 106,625.47 lbs

Step 6: Calculate new buoyant force
(FBSLAB) = $\rho_w *$ Slab volume = 62.4 lbs/ft$^3$ x 105.72 ft$^3$ = 6,596.93 ft$^3$
Fb = 88,833.83 lbs. + 6,596.93 lbs. = 95,430.76 lbs

Step 7: Calculate new FS
FS = 106,625.47 lbs / 95,430.76 lbs
FS = 1.12 > 1.1 OK
Yes, we could make the slab 10 in. instead of 1 ft.